Tentamen Signal Analysis, 5/2/04, room 13.202, 9.00-12.00

Please write your name and student number on each sheet; answer either in Dutch or English.

Question 1

a. Calculate the Fourier transforms of the functions $f(t)=\sin(2\pi f_0 t)$ and $g(t)=e^{-t/\tau}$ for $t\geq 0$ and $g(t)=0$ for $t<0$.

b. Calculate the convolution $f*g$ by using the result of **a**.

c. Draw $f*g$ for $\tau << 1/f_0$ and for $\tau > 1/f_0$, and explain this result using a graphical representation of the convolution operation.

Question 2

a. A sequence $s(n)$ is defined by $s(-2)=-3$, $s(-1)=2$, $s(0)=-1$, $s(1)=2$, $s(2)=3$, and $s(i)=0$ for $i \notin \{-2,-1,0,1,2\}$. Calculate the Fourier transform $\hat{S}(f)$ of $s(n)$, and write it as a simple sum of a DC-term, a sine and a cosine.

b. The sequence of **a** is input to a digital filter characterized by $w(n)+\alpha\, w(n-1)+\beta w(n-2)=s(n)+\gamma\, s(n-1)$, where $w(n)$ is the resulting output sequence. Find the Fourier transform $\hat{W}(f)$ of the output.

c. Sketch a physical realization of the filter defined in **b** in terms of amplifiers and delay lines.

Question 3

The joint probability density function (pdf) of two random variables *x* and *y* is given by $p(x,y)=1/\pi$ for $x^2+y^2\leq 1$, and $p(x,y)=0$ elsewhere.

a. Show that the marginal pdf $p(x)$ is given by $p(x)=\frac{2}{\pi}\sqrt{1-x^2}$ for $-1\leq x\leq 1$, and $p(x)=0$ elsewhere. Make a sketch of $p(x,y)$, $p(x)$, and $p(y)$.

b. Calculate the conditional pdf $p(y\,|\,x)$.

c. Show whether *x* and *y* are independent.

d. Calculate the covariance of *x* and *y*.

Question 4

The random signal $s(t)$ is the sum of two statistically independent, stationary random signals $x(t)$ and $y(t)$, where $x(t)$ has an autocorrelation function $R_x(\tau)=\exp(-\alpha\tau^2)$, with α a positive constant; $y(t)$ is zero mean white noise with the power spectral density $\mathsf{P}_y(f)=A$. This signal $s(t)=x(t)+y(t)$ is passed through a linear filter with impulse response $h(t)$

$$\begin{aligned} h(t) &= \frac{1}{\sigma}e^{-t/\sigma} && \text{for } t\geq 0 \\ h(t) &= 0 && \text{for } t<0 \end{aligned}.$$

a. Calculate the power spectral density of signal $x(t)$. (You may want to use $\int_{-\infty}^{\infty} e^{-ct^2}dt=\sqrt{\frac{\pi}{c}}$)

b. Calculate the autocorrelation function and power spectral density of $s(t)$.

c. Find the power spectral density of the output signal